where $S(I, J)$ is the score for subsequences I and J and $S(\emptyset, \emptyset) = 0$. Elements of the alignment matrix are $M_{i,j}$, and since we are not applying indel penalties at the ends of A and B, we write

$$M_{i,0} = M_{0,j} = 0. \tag{6.13}$$

The score up to and including the matrix element $M_{i,j}$ is calculated by using scores for the elements immediately above and to the left (including the diagonal), but this time scores that fall below zero will be replaced by zero. The scoring for matches, mismatches, and indels is otherwise the same as for global alignment. The resulting expression for scoring $M_{i,j}$ is

$$M_{i,j} = \max \left\{ \begin{array}{l} M_{i-1,j-1} + s(a_i, b_i) \\ M_{i-1,j} - \delta \\ M_{i,j-1} - \delta \\ 0 \end{array} \right\}. \tag{6.14}$$

The best local alignment is the one that ends in the matrix element having the highest score:

$$\max\{S(I, J) : I \subset A, J \subset B\} = \max_{i,j} M_{i,j}. \tag{6.15}$$

Thus, the best local alignment score for strings A and B is

$$M(A, B) = \max_{i,j} M_{i,j}. \tag{6.16}$$

Computational Example 6.2: Local alignment

Determine the best local alignment and the maximum alignment score for A = `ACCTAAGG` and B = `GGCTCAATCA`. For scoring, take $s(a_i, b_j) = +2$ if $a_i = b_j$, $s(a_i, b_j) = -1$, $a_i \neq b_j$, and $s(a_i, -) = s(-, b_j) = -2$.

Step 1: Write down the alignment matrix using B along the top and A in a column at the side.

Step 2: Fill in the first row and first column by using (6.13).

Step 3: Then fill in all matrix elements using the scoring rule (6.14), keeping track of the paths into each element. For clarity, we have included below only the arrows around the highest-scoring path. Observe what happens for $M_{3,7}$. Regardless of whether this is entered from above, from the left, or diagonally from the left, the scoring rule would have yielded -1 were it not for the requirement that all elements be non-negative, as indicated in (6.14).

Step 4: The local alignment ends at $M_{6,7}$ (shaded box), which contains the maximum alignment score (6). Read out the alignment, starting at $M_{6,7}$ and working backward along the directions of entry into each cell until an element containing zero is encountered.