

1 if there had been no breakpoint). Using the same reasoning, there is also a breakpoint after the last element. In fact, $b(G) = 8$, and $d(G, I) \geq 8/2$. We know that it will take at least four reversals to transform G to I . In the illustration, we employed five reversals.

What is the greatest number of reversals required to transform the most unfavorable permutation G^* to I (worst-case scenario)? What is the least number of reversals it would take to transform G^* to I ? We can easily estimate what the greatest number of reversals (worst case) would be. We can imagine that you start from this worst-case permutation by reversing the set of elements from the initial one in the permutation up to and including g_1 . This puts g_1 in front. Then reverse all elements beginning with the second in the permutation up to and including g_2 . Continue the process until the identity permutation is achieved. In the worst case, we will have to do this for $n - 1$ elements. We don't have to do a separate operation for g_n because if the first $n - 1$ elements are sorted as described, then n automatically falls into the last position. It can be shown that, for some permutations, the *least* number of reversals required is $(n + 1)/2$. Thus we have for the worst case G^*

$$(n + 1)/2 \leq d(G^*, I) \leq n - 1.$$

Examples of the worst case are

$$G_{13}^* = 3\ 1\ 5\ 2\ 7\ 4\ 9\ 6\ 11\ 8\ 12\ 10\ 13$$

and

$$G_{14}^* = 3\ 1\ 5\ 2\ 7\ 4\ 9\ 6\ 11\ 8\ 13\ 10\ 12\ 14.$$

5.2.2 Estimating Reversal Distances by Cycle Decomposition

Given X and Y , which are permutations of the same letters, how can we determine $d(X, Y)$? We describe a graphical method for doing this, and as an example (to keep the graphs simple) we estimate the reversal distance between F and I , where

$$F : \quad 1\ 2\ 4\ 5\ 3 \qquad I : \quad 1\ 2\ 3\ 4\ 5.$$

First, extend each permutation by adding 0 before the first element and $n + 1$ after the last element, where n is the number of elements. These changes do not alter the reversal distances between the permutations because the elements 0 and $n + 1$ are in their correct positions.

$$F' : \quad 0\ 1\ 2\ 4\ 5\ 3\ 6 \qquad I' : \quad 0\ 1\ 2\ 3\ 4\ 5\ 6.$$

Each element index corresponds to a **vertex** of the graph, and lines connecting the vertices are called **edges**. First, we connect all adjacent vertices of F' with black edges:

$$F' : \quad 0 \text{ — } 1 \text{ — } 2 \text{ — } 4 \text{ — } 5 \text{ — } 3 \text{ — } 6$$

Now, in the same way, connect all of the adjacent I' vertices with grey edges.